$$\left(\frac{\partial P}{\partial V}\right)_S = \frac{dP_S}{dV} , \left(\frac{\partial P}{\partial V}\right)_H = \frac{dP_H}{dV}$$

$$\left(\frac{\partial E}{\partial V}\right)_H = \frac{dE_H}{dV} , \left(\frac{\partial E}{\partial V}\right)_S = \frac{dE_S}{dV} = -P_S . \tag{36}$$

Equation (35) then simplifies to

$$\frac{dP_S}{dV} + kP_S = \frac{d}{dV}(P_H - kE_H) . \tag{37}$$

The pressure and energy on the Hugoniot are expressed as

$$P_H = \frac{C^2 a}{(V_0 - Ma)^2}$$

$$E_H = P_H a/2$$

when $a = V_0 - V$ is substituted into Eqs. (6) and (32). Substitution of derivatives of P_H and E_H with respect to a into Eq. (37) yields

$$\frac{dP_S}{da} - kP_S = \frac{C^2}{(V_0 - Ma)^3}\left[V_0 + a(M - kV_0)\right] . \tag{38}$$

This first order differential equation can be solved using the integrating factor $\exp(\int k da)$. Hence,

$$P_S = Ae^{ka} + e^{ka} \int e^{-ka} C^2 \left[\frac{V_0 + a(M - kV_0)}{(V_0 - Ma)^3}\right] da \tag{39}$$

where A is a constant of integration. The integral term can be performed in a never ending series of integrations by parts, but an easier method using information gained from integrating by parts is to assume a series solution of the form

$$P_S = Ae^{ka} + \frac{C^2}{(V_0 - Ma)^2} \sum_{i=0}^{\infty} A_i a^i . \tag{40}$$

The A_i's must be chosen such that Eq. (38) is satisfied for all powers of a. The recursion relation for the A_i's which satisfies this requirement is